Gate tunable spin transport in graphene with Rashba spin-orbit coupling

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Abstract

Recently, it attracts much attention to study spin-resolved transport properties in graphene with Rashba spin-orbit coupling (RSOC). One remarkable finding is that Klein tunneling in single layer graphene (SLG) with RSOC (SLG + R for short below) behaves as in bi-layer graphene (BLG). Based on the effective Dirac theory, we reconsider this tunneling problem and derive the analytical solution for the transmission coefficients. Our result shows that Klein tunneling in SLG + R and BLG exhibits completely different behaviors. More importantly, we find two new transmission selection rules in SLG + R, i.e., the single band to single band (S/S) and the single band to multiple bands (S/M) transmission regimes, which strongly depend on the relative height among Fermi level, RSOC, and potential barrier. Interestingly, in the S/S transmission regime, only normally incident electrons have capacity to pass through the barrier, while in the S/M transmission regime the angle-dependent tunneling becomes very prominent. Using the transmission coefficients, we also derive spin-resolved conductance analytically, and conductance oscillation with the increasing barrier height and zero conductance gap are found in SLG + R. The present study offers new insights and opportunities for developing graphene-based spin devices.

1. Introduction

Spin transport and control are of fundamental and practical importance in future graphene-based spintronic devices [1]. Klein tunneling is one of the most supernatural and counterintuitive transport phenomena in which an incoming electron can penetrate through a potential barrier even if its energy is lower than the barrier height [2]. Observation of this exotic phenomenon is quite difficult, because it requires extremely high energy in experiment [3]. Graphene, a single atomic layer of graphitic carbon, exhibits the linear spectrum near the Dirac points, where the carriers behave like massless Dirac fermions. This unique property makes it possible to test the relative quantum tunneling in graphene experimentally. Thus, it arouses...
intense interest to investigate Klein tunneling in graphene-based systems [4–13]. Most theoretical works focused mainly on the case of spin-independent tunneling, while spin-orbit coupling (SOC) was less considered.

The SOC in graphene comprises intrinsic and extrinsic components, which play a key role in generating a topological insulating state and manipulating electron spins. However, the intrinsic SOC in graphene is extremely weak. Thus, various methods to induce the large extrinsic SOC have been proposed theoretically [14–16] and explored experimentally [17–19]. Rashba SOC (RSOC) originating from the space inversion symmetry breaking, has attracted tremendous research interest in various field of physics and materials science [20], owing to its controllability via an external gate voltage [21]. Marchenko et al. reported a large RSOC in the Au-intercalated graphene-Ni system [17]. Such enhancement is crucial for the development of graphene-based devices such as the Das-Datta spin field effect transistor (FET) [22]. Motivated by the experimental development, recently much attention has paid to investigate the effect of RSOC on spin transport properties of graphene [23–28]. Among these works, one remarkable finding was that Klein tunneling in single layer graphene (SLG) with RSOC (SLG + R for short below) behaves as in bi-layer graphene (BLG) [25]. Considering the early work by Katsnelson et al. [4], such fantastic behavior seems somewhat surprising.

As demonstrated in Ref. [4], chiral tunneling in SLG and BLG exhibits completely different behaviors. So, why does the appearance of RSOC re-paint a new tunneling picture in SLG? Admittedly, the low-energy spectrum of SLG is independent of the momentum. The parameter \( \gamma_1 \) in BLG and the Rashba coupling \( \lambda \) in SLG are two fundamentally different interactions. RSOC in SLG comes from the \( \pi-\sigma \) hybridization [29], and unlike conventional semiconduction 2D electron gases, it is independent of the momentum. The parameter \( \gamma_1 \) in BLG characterizes the nearest-neighbor hopping between the two graphene layers, and almost has no effect on SOC as demonstrated in Refs. [30,31]. Furthermore, to the best of our knowledge, carriers in BLG are massive and similar to conventional non-relativistic quasiparticles [32], which are quite different from massless Dirac fermions in SLG. In fact, it requires different equations to describe electrons in BLG and SLG + R. As a result, for tunneling problems in BLG, there are four possible solutions for a given energy, and two of them correspond to evanescent waves [4] which do not exist in SLG + R. Combining these facts, we think that it is necessary to reexamine whether Klein tunneling in SLG + R behaves as in BLG.

In this paper, we reconsider the tunneling problem in SLG + R based on the effective Dirac theory. We show that the tunneling in SLG + R and BLG exhibits different behaviors. In BLG, normally incident electrons are always completely reflected by the potential barrier [4], but this is not always true for SLG + R case. In SLG + R, normally incident electrons are allowed some tunneling, but are not always perfectly transmitted, consistent with the previous report in Ref. [23]. Interestingly, when the Fermi lever in SLG + R is less than the strength of RSOC \( (0 < E_F < \lambda) \), the tunneling only occurs in normally incident electrons. We analyze this case and obtain the analytical solution for the transmission coefficient, revealing the properties of spin polarization independent of the height of potential barrier. We also discuss the case of \( E_F > \lambda \) in more detail, and intriguing behaviors of the tunneling and the spin-resolved conductance oscillations will be shown.

This paper is organized as follows. In Sec. 2 we describe our model and solve it analytically. In Sec. 3 we investigate the subband-dependent transmission coefficients in SLG + R. Then, we derive and discuss the spin-resolved conductance in Sec. 4. In Sec. 5 we discuss the dependence of spin polarization on the barrier height. Based on the spin transport properties discussed above, in Sec. 6 we propose a conceptive graphene-spin device. Our conclusions are summarized in Sec. 7.

### 2. Model

Near the Dirac points \( K \) and \( K' \), the effective Hamiltonian of SLG + R is described by Ref. [33].

\[
H^0_F = -i\hbar v_F \left( \tau \sigma_y \frac{\partial}{\partial x} + \sigma_y \frac{\partial}{\partial y} \right) + \frac{1}{2} \lambda (\tau \sigma_y S_y - \sigma_x S_x), 
\]

(1)

where \( S_i \) and \( \sigma_i \) \( (i = x, y) \) are Pauli matrices denoting the real spin and the pseudospin, respectively. \( v_F \approx 10^6 \text{m/s} \) is the Fermi velocity, and \( \lambda \) represents RSOC strength. \( \tau = \pm 1 \) describes two inequivalent Dirac cones \( (K \text{ and } K') \). Considering the RSOC, the electronic bands near \( K \) point are given by

\[
E_{\nu \mu} = \frac{\nu \times \mu}{2} \left[ \sqrt{\lambda^2 + 4(\hbar v_F k)^2} - \mu \lambda \right], \quad \nu, \mu = \pm 1,
\]

(2)

where the product \( \nu \times \mu \) specifies electron and hole subbands. The corresponding eigenstates are

\[
|\nu \mu\rangle = \frac{1}{\sqrt{2(1 + \chi_{\nu \mu})}} \left[ \begin{array}{c} \nu e^{-i\phi} \\ \chi_{\nu \mu} e^{-i\phi} \\ \nu \chi_{\nu \mu} e^{-i\phi} \\ 1 \end{array} \right],
\]

(3)
with \( \phi = \arctan(k_x/k_y) \), \( \chi_{\mu} = E_{\mu}/(\hbar v_F) \), and \( k = \sqrt{(k_x^2 + k_y^2)} \). Here \( k_x \) and \( k_y \) are the Cartesian components of electron wave vector measured from \( K \) point. The spin averages for state \( |\mu\rangle \) are given by

\[
\langle S_x \rangle_{\mu} = \frac{2 \mu \hbar v_F k \sin(\phi)}{\sqrt{\lambda^2 + 4(\hbar v_F k)^2}}, \quad \langle S_y \rangle_{\mu} = \frac{-2 \mu \hbar v_F k \cos(\phi)}{\sqrt{\lambda^2 + 4(\hbar v_F k)^2}}, \quad \langle S_z \rangle_{\mu} = 0,
\]

(4)

which are independent of \( \nu \) and lie in the graphene plane. Clearly, for a given momentum, the real spin of an electron occupying the state \( |\nu, \mu = 1\rangle \) is in the opposite polarization direction to the electron occupying the state \( |\nu, \mu = -1\rangle \). Moreover, the in-plane spin is perpendicular to the electron momentum because of \( \langle S \cdot K \rangle_{\mu} = 0 \), as shown in Fig. 1(a).

Considering an external potential barrier,

\[
V(x) = \begin{cases} U, & 0 < x < \Delta \\ 0, & \text{otherwise} \end{cases}
\]

(5)

then the system Hamiltonian is described by

\[
H_x = H_0^x + V(x).
\]

(6)

Here, we assume that the sample of SLG with length \( L \) and width \( W \) is big enough that boundary effect can be ignored. Thus, for the \( K \)-point Hamiltonian (6), it has the following formal solutions in regions I, II, and III [see Fig. 1(b)], respectively,

\[
\psi^I(x, y) = \frac{1}{\sqrt{2LW(1 + \eta^2_1)}} \left[ \frac{i(-1)^{n+1}e^{i2\theta_1}}{1} \right] e^{i(k_1x + k_1y)} + \frac{r_1}{\sqrt{2LW(1 + \eta^2_2)}} \left[ \frac{ie^{i2\theta_1}}{-\eta_1 e^{i\theta_1}} \frac{-\eta_1 e^{i\theta_1}}{1} \right] e^{i(-k_1x + k_1y)}
\]

\[
\psi^II(x, y) = \frac{\alpha_1}{\sqrt{2LW(1 + \xi^2_{15})}} \left[ \frac{ie^{-i2\theta_1}}{1} \right] e^{i(q_1 x + k_1 y)} + \frac{\beta_1}{\sqrt{2LW(1 + \tau^2_{15})}} \left[ \frac{ie^{i2\theta_1}}{-\xi_{15} e^{i\theta_1}} \frac{-\xi_{15} e^{i\theta_1}}{1} \right] e^{i(-q_1 x + k_1 y)}
\]

\[
\psi^III(x, y) = \frac{\alpha_2}{\sqrt{2LW(1 + \xi^2_{25})}} \left[ \frac{-ie^{-i2\theta_2}}{1} \right] e^{i(q_2 x + k_1 y)} + \frac{\beta_2}{\sqrt{2LW(1 + \tau^2_{25})}} \left[ \frac{-ie^{i2\theta_2}}{-\xi_{25} e^{i\theta_2}} \frac{-\xi_{25} e^{i\theta_2}}{1} \right] e^{i(-q_2 x + k_1 y)}
\]

(7)

\[
(8)
\]

![Fig. 1](https://example.com/fig1.png)

Fig. 1. (a) The low-energy band structure of SLG + R, and the spin configuration of electrons with arrows marking the spin directions. Blue and red lines denote \( \nu = -1 \) and \( \nu = 1 \) subbands, respectively. The spin of an electron occupying the different \( \nu \)-subband has the opposite orientation, and is perpendicular to the electron momentum. (b) The tunable potential barrier with height \( U \) and width \( \Delta \). The Fermi level \( E_F \) is denoted by the purple dashed line. The azury filled areas indicate occupied states. [For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.]
\[
\psi^K_{\text{III}}(x, y) = \frac{t_1}{\sqrt{2LW(1 + \eta_1^2)}} \begin{bmatrix}
-\frac{i e^{-i\theta_1}}{\eta_1 e^{-i\phi_1}} \\
\frac{\eta_1 e^{-i\phi_1}}{1}
\end{bmatrix} e^{i(k_{1x}x + k_{1y}y)} + \frac{t_2}{\sqrt{2LW(1 + \eta_2^2)}} \begin{bmatrix}
-\frac{i e^{-i\theta_2}}{\eta_2 e^{-i\phi_2}} \\
\frac{\eta_2 e^{-i\phi_2}}{1}
\end{bmatrix} e^{i(k_{2x}x + k_{2y}y)},
\]

(9)

where

\[
\theta_n = \arcsin \left( \frac{k_{nx}}{q_n} \right), \quad \phi_n = \arcsin \left( \frac{k_{ny}}{k_n} \right),
\]

(10)

\[
\eta_n = \frac{E_n}{\hbar v_F k_n}, \quad \xi_n = \frac{\Omega_{ns}}{\hbar v_F q_n}.
\]

(11)

\[
k_{nx} = \sqrt{k_n^2 - q_n^2}, \quad q_{nx} = \sqrt{q_n^2 - k_n^2},
\]

(12)

\[
k_n = \frac{1}{\hbar v_F} \sqrt{E - (-1)^n \lambda}, \quad \text{(wave vector outside the barrier)}
\]

(13)

\[
q_n = \frac{1}{\hbar v_F} \sqrt{(E - U)(E - U - (-1)^n \lambda)}, \quad \text{(wave vector inside the barrier)}
\]

(14)

\[
E_n = \frac{1}{2} \left[ \sqrt{\lambda^2 + 4(\hbar v_F k_n)^2} + (-1)^n \lambda \right],
\]

(15)

\[
\Omega_{ns} = \frac{1}{2} \left[ s \sqrt{\lambda^2 + 4(\hbar v_F q_n)^2} + (-1)^n \lambda \right],
\]

(16)

with \( s = \text{sign}(E - U) \), \( n = 1, 2 \) describe two different spin-split subbands as indicated in Fig. 1(a).

Similarly, for the \( K' \) valley the wave functions in regions I, II, and III [see Fig. 1(b)], have the following forms:

\[
\psi^K_{I}(x, y) = \frac{1}{\sqrt{2LW(1 + \eta_1^2)}} \begin{bmatrix}
\frac{i(-1)^{n+1} \eta_{1x} e^{-i\phi_1}}{1} \\
\frac{i(-1)^n \eta_{1x} e^{-i\phi_1}}{-\eta_{1y} e^{-i\phi_1}}
\end{bmatrix} e^{i(k_{1x}x + k_{1y}y)} + \frac{r_1}{2LW(1 + \eta_1^2)} \begin{bmatrix}
\frac{-i \eta_{1y} e^{i\phi_1}}{1} \\
\frac{-i \eta_{1x} e^{i\phi_1}}{\eta_{1y} e^{i\phi_1}}
\end{bmatrix} e^{i(-k_{1x}x + k_{1y}y)},
\]

(17)

\[
\psi^K_{II}(x, y) = \frac{\alpha_1}{\sqrt{2LW(1 + \eta_1^2)}} \begin{bmatrix}
\frac{i \xi_{1y} e^{-i\phi_1}}{1} \\
\frac{-i \xi_{1y} e^{-i\phi_1}}{-\xi_{1x} e^{-i\phi_1}}
\end{bmatrix} e^{i(q_{1x}x + q_{1y}y)} + \frac{\beta_1}{\sqrt{2LW(1 + \eta_1^2)}} \begin{bmatrix}
\frac{-i \xi_{1x} e^{i\phi_1}}{1} \\
\frac{-i \xi_{1x} e^{i\phi_1}}{\xi_{1y} e^{i\phi_1}}
\end{bmatrix} e^{i(-q_{1x}x + q_{1y}y)} + \frac{\alpha_2}{\sqrt{2LW(1 + \eta_2^2)}} \begin{bmatrix}
\frac{-i \xi_{2x} e^{-i\phi_2}}{1} \\
\frac{-i \xi_{2x} e^{-i\phi_2}}{-\xi_{2y} e^{-i\phi_2}}
\end{bmatrix} e^{i(q_{2x}x + q_{2y}y)} + \frac{\beta_2}{\sqrt{2LW(1 + \eta_2^2)}} \begin{bmatrix}
\frac{i \xi_{2y} e^{i\phi_2}}{1} \\
\frac{i \xi_{2y} e^{i\phi_2}}{\xi_{2x} e^{i\phi_2}}
\end{bmatrix} e^{i(-q_{2x}x + q_{2y}y)},
\]

(18)

\[
\psi^K_{III}(x, y) = \frac{t_1}{\sqrt{2LW(1 + \eta_1^2)}} \begin{bmatrix}
\frac{i \eta_{1x} e^{-i\phi_1}}{1} \\
\frac{-i \eta_{1x} e^{-i\phi_1}}{-\eta_{1y} e^{-i\phi_1}}
\end{bmatrix} e^{i(k_{1x}x + k_{1y}y)} + \frac{t_2}{\sqrt{2LW(1 + \eta_2^2)}} \begin{bmatrix}
\frac{-i \eta_{2y} e^{i\phi_2}}{1} \\
\frac{-i \eta_{2y} e^{i\phi_2}}{-\eta_{2x} e^{i\phi_2}}
\end{bmatrix} e^{i(k_{2x}x + k_{2y}y)},
\]

(19)

where the specific parameters are equal to the above definitions as listed in Eqs. 10–16.
3. Klein tunneling in SLG + R

Now, let us discuss in detail the role of RSOC in the tunneling problem. We define $T_{nm}$ ($n, m = 1, 2$) as the tunneling probability of an electron initial in subband $n$ and outgoing in subband $m$. Then, the total transmission ability of each incident electron is given by

$$T_n^{\text{tot}} = T_{n1} + T_{n2}, \quad n = 1, 2$$  \hspace{1cm} (20)

Note that the Hamiltonian (6) obeys the time-reversal symmetry. Thus, the transmission coefficients are identical for $K$ and $K'$ valleys. Indeed, straightforward calculations also show that the unsettled parameters in wave functions for $K$ and $K'$ valleys, satisfy the same equation set (A1), as shown in the Appendix A. So, in the following we only need to discuss the $K$ valley.

3.1. The case of $0 < E_F < \lambda$

First, we consider the case of $0 < E_F < \lambda$. According to the boundary conditions that require the continuity of wave function, the transmission coefficients $T_1^{\text{tot}}$ and $T_2^{\text{tot}}$ in this case are calculated as

$$T_1^{\text{tot}}(E, \lambda, U, \Delta, \phi_1) = \begin{cases} 
\tilde{T}_{11} & (\phi_1 = 0, \ 0 < U < E_F) \\
0 & (\phi_1 = 0, \ E_F < U < E_F + \lambda) \\
\tilde{T}_{11} & (\phi_1 = 0, \ U > E_F + \lambda) \\
0 & (-\pi/2 \leq \phi_1 < 0, \ 0 < \phi_1 \leq \pi/2, \ U > 0)
\end{cases}$$ \hspace{1cm} (21)

$$T_2^{\text{tot}}(E, \lambda, U, \Delta, \phi_2) = 0, \ (-\pi/2 \leq \phi_2 \leq \pi/2, \ U > 0),$$ \hspace{1cm} (22)

where $\tilde{T}_{11}$ is given by

$$\tilde{T}_{11} = \left[1 + \frac{1}{4}(\rho_1 + 1/\rho_1 - 2)\sin^2(\Theta_1)\right]^{-1},$$ \hspace{1cm} (23)

with

$$\rho_1 = \frac{E(E - U + \lambda)}{(E - U)(E + \lambda)},$$ \hspace{1cm} (24)

$$\Theta_1 = \frac{\Delta}{\hbar_{\text{HF}}} \sqrt{(E - U)(E - U + \lambda)}.$$ \hspace{1cm} (25)

It deserves to be mentioned that in this case only normal incident electrons have capacity to pass through the barrier. Moreover, the electrons always occupy the same subband before and after penetrating through the barrier, thus suggesting a single band to single band ($S \to S$) transmission regime. This special tunneling behavior is very different from the case without considering RSOC [4]. In Ref. [4], it was shown that the transmission probability of an electron depends on incident angles at a given potential barrier. Moreover, the tunneling at normal incidence is always 100%. However, in the presence of RSOC, only if the resonance conditions ($\Theta_1 = N\pi, N = 0, 1, \ldots$) are satisfied, can normally incident electrons penetrate through the barrier perfectly according to Eq. (23). Therefore, RSOC plays a significant role in the $S \to S$ transmission regime.

3.2. The case of $E_F > \lambda$

Then, let us discuss the case of $E_F > \lambda$. In this case, the tunneling depends not only on the $S \to S$ transmission regime but also on the single band to multiple bands ($S \to M$) transmission regime. Thus, the transmission should be discussed in five different regions according to the barrier height. In the $S \to M$ transmission regime, the subband-dependent transmission coefficients are limited by the critical incident angle $\phi_0^c$. If $|\phi_0| \geq \phi_0^c$, we have $T_{nm} = 0$ ($n, m = 1, 2$), while $|\phi_0| < \phi_0^c$ the transmission coefficients are calculated as

$$T_{11} = \frac{\det(H_{11} - C^{-1}M)}{\det(D - C^{-1}B)}^2,$$ \hspace{1cm} (26)

$$T_{12} = \frac{\eta_2 \cos(\phi_2)}{\eta_1 \cos(\phi_1)} \left|\frac{\det(H_{12} - C^{-1}F)}{\det(D - C^{-1}B)}\right|^2,$$ \hspace{1cm} (27)
\[ T_{21} = \frac{\eta_1 \cos(\phi_1)}{\eta_2 \cos(\phi_2)} \left| \frac{\det(H_{21} - CA^{-1}M)}{\det(D - CA^{-1}B)} \right|^2, \tag{28} \]
\[ T_{22} = \left| \frac{\det(H_{22} - CA^{-1}F)}{\det(D - CA^{-1}B)} \right|^2, \tag{29} \]

where the specific parameters are shown in the Appendix A.

i) For \( 0 < U < E_F - \lambda \) (\( S \rightarrow M \) transmission regime),

\[ T^\text{tot}_{1}(E, \lambda, U, \Delta, \phi_1) = \begin{cases} 0, & \pi/2 \geq |\phi_1| \geq \phi_1^* = \arcsin\left(\frac{(E - U)(E - U - \lambda)}{E(E + \lambda)}\right), \\ T_{11} + T_{12}, & \text{otherwise} \end{cases} \tag{30} \]
\[ T^\text{tot}_{2}(E, \lambda, U, \Delta, \phi_2) = \begin{cases} 0, & \pi/2 \geq |\phi_2| \geq \phi_2^* = \arcsin\left(\frac{(E - U)(E - U - \lambda)}{E(E + \lambda)}\right), \\ T_{21} + T_{22}, & \text{otherwise} \end{cases} \tag{31} \]

ii) For \( E_F - \lambda < U < E_F \) (\( S \rightarrow S \) transmission regime),

\[ T^\text{tot}_{1}(E, \lambda, U, \Delta, \phi_1) = \begin{cases} \tilde{T}_{11}, & \phi_1 = 0 \\ 0, & \text{otherwise} \end{cases}, \tag{32} \]
\[ T^\text{tot}_{2}(E, \lambda, U, \Delta, \phi_2) = 0, \quad (-\pi/2 \leq \phi_2 \leq \pi/2), \tag{33} \]

where \( \tilde{T}_{11} \) is given by Eq. (23).

iii) For \( E_F < U < E_F + \lambda \) (\( S \rightarrow S \) transmission regime),

\[ T^\text{tot}_{1}(E, \lambda, U, \Delta, \phi_1) = 0, \quad (-\pi/2 \leq \phi_1 \leq \pi/2), \tag{34} \]
\[ T^\text{tot}_{2}(E, \lambda, U, \Delta, \phi_2) = \begin{cases} \tilde{T}_{22}, & \phi_2 = 0 \\ 0, & \text{otherwise} \end{cases}, \tag{35} \]

where

\[ \tilde{T}_{22} = \left[ 1 + \frac{1}{4}(\rho_2 + 1/\rho_2 - 2)\sin^2(\Theta_2) \right]^{-1}, \tag{36} \]

with

\[ \rho_2 = \frac{E(E - U - \lambda)}{(E - U)(E - \lambda)}, \tag{37} \]
\[ \Theta_2 = \frac{\Delta}{\hbar v_F} \sqrt{(E - U)(E - U - \lambda)}. \tag{38} \]

iv) For \( E_F + \lambda < U < 2E_F \) (\( S \rightarrow M \) transmission regime),
Transmissions within subband discuss the nontrivial region of remarkable. We can see in Fig. 3(a) that the small-angle-incident electrons prefer tunneling with a probability the insets of Fig. 2(a) and (b). When we increase the barrier height to $T_{tot}$ is very weak, and probabilities in the range of about unlike the discontinuous behavior of as shown in Fig. 2(b). When the incident angle is toward to the critical value is comparable with $12\ (top)$ and $17\ (bottom)$. When an incoming electron occupying the subband $T_{tot}$ plays a preponderant role in the tunneling. Moreover, the total transmission probability almost reaches transmission regime ($S \rightarrow M$ transmission regime),

$$T_{tot}^1(E, \lambda, U, \Delta, \phi_1) = \begin{cases} 0, & \pi/2 \geq |\phi_1| \geq \phi_1^c = \arcsin \left( \frac{\sqrt{(E - U)(E - U + \lambda)}}{E + \lambda} \right), \\
T_{11} + T_{12}, & \text{otherwise} \end{cases}$$

(39)

$$T_{tot}^2(E, \lambda, U, \Delta, \phi_2) = \begin{cases} 0, & \pi/2 \geq |\phi_2| \geq \phi_2^c = \arcsin \left( \frac{\sqrt{(E - U)(E - U + \lambda)}}{E - \lambda} \right), \\
T_{21} + T_{22}, & \text{otherwise} \end{cases}$$

(40)

According to Eqs. (30), (31), (39)–(42), we have $T_{tot}^n = 0$ when $|\phi_n| \geq \phi_n^c$ ($n = 1, 2$). So, in the following we only need to discuss the nontrivial region of $|\phi_n| < \phi_n^c$ ($n = 1, 2$). Figs. 2–4 show examples of angle- and subband-dependent transmission probabilities in the $S \rightarrow M$ transmission regime. Here, the Fermi energy of incident electrons is chosen as $E_F = 80$ meV that is most typical in experiments with graphene [4]. When $U = 15$ meV, as can be seen in Fig. 2(a), the transmission probability $T_{12}$ is very weak, and $T_{11}$ plays a preponderant role in the tunneling. Moreover, the total transmission probability almost reaches $T_{tot}^1 = 1$ for all incident angles within $|\phi_1| < \phi_1^c = 17.83^\circ$. By contrast, $T_{22}$ strongly depends on the incident angle of electron, as shown in Fig. 2(b). When the incident angle is toward to the critical value $\phi_2^c = 39.6^\circ$, $T_{22}$ decreases fast, and only in the range of about $|\phi_2| < 30^\circ$, $T_{22}$ is comparable with $T_{12}$. We can also see that $T_{12}$ and $T_{21}$ display the similar behavior, as shown in the insets of Fig. 2(a) and (b). When we increase the barrier height to $U = 145$ meV, the $S \rightarrow M$ transmission becomes remarkable. We can see in Fig. 3(a) that the small-angle-incident electrons prefer tunneling with a probability $T_{11}$. When the incident angle is located in the interval of about $9.3 < |\phi_1| < 12.8^\circ$, we can see $T_{12} > T_{11}$, suggesting that there is no simple dominant relation between $T_{11}$ and $T_{12}$. When an incoming electron occupying the subband $n = 2$, it is prone to penetrate through the barrier with a probability $T_{22}$, but $T_{21}$ holds an advantage over $T_{22}$ near the $\phi_2^c = 39.6^\circ$ [see Fig. 3(b)]. In addition, unlike the discontinuous behavior of $T_{11}$ and $T_{12}$ ($T_{21}$ and $T_{22}$) at the critical angle $\phi_1^c = 17.83^\circ$ ($\phi_2^c = 39.6^\circ$) [see Figs. 2 and 3].

![Fig. 2. Angle- and subband-dependent transmissions through a barrier of wide $\Delta = 110$ nm and height $U = 15$ meV in SLG + R with $E_F = 80$ meV and $\lambda = 50$ meV.](image)

(a) Transmissions within $|\phi_1| < \phi_1^c = 17.83^\circ$ for the incident electron in subband $n = 1$. (b) Transmissions within $|\phi_2| < \phi_2^c = 39.6^\circ$ for the incident electron in subband $n = 2$. The insets show $T_{12}$ (top) and $T_{22}$ (bottom).
the corresponding transmission coefficients for $U = 200$ meV are continuous at the critical angle $\phi_1^c = 28.71^\circ$ (\(\phi_2^c = 90^\circ\)), because the transmission probabilities just vanish at this critical incident angle (see Fig. 4).

In order to further reveal the influences of angle ($\phi_n$), subbands ($n = 1, 2$), and potential barrier ($U$) on transmission probabilities, we depict $T_{11}^{tot}$, $T_{12}$, and $T_{22}$ ($n = 1, 2$) as a function of $U$ and $\phi_n$ in Fig. 5, which intuitively exhibits the global distributions of the transmission probabilities at a given $\lambda = 50$ meV. For lower barrier ($0 < U < 30$ meV) electrons in subband $n = 1$ almost perfectly pass through the barrier for all incident angles within $|\phi_1| < \phi_1^c$ [see Fig. 5(a)]. When the barrier is located in the range of $30$ meV $< U < 80$ meV, only normal incident electrons are allowed tunneling with a probability $T_{11}$ [see Fig. 5(b)]. When $80$ meV $< U < 130$ meV, the transmissions are forbidden for electrons in subband $n = 1$, but only the normally incident electrons occupying the subband $n = 2$ are allowed tunneling with a probability $T_{22}$ [see Fig. 5(c)]. For a higher barrier ($130$ meV $< U < 200$ meV) tunneling is very sensitive to the small-angle incidence, where $T_{11}$ plays a significant role [see Fig. 5(b)], compared with $T_{12}$ [see Fig. 5(c)]. We also see in Fig. 5(c) that $T_{12}$ has no contribution to $T_{11}^{tot}$ in the case of $0 < U < 130$ meV, where the similar behaviors are also found in $T_{21}$ [see Fig. 5(f)]. When it comes to $T_{22}$ [see Fig. 5(e)], similar to $T_{11}$, it is the main contributor to total transmission probability $T_{22}^{tot}$, compared with $T_{21}$. So, on the whole, the electrons prefer tunneling between the same spin-split subbands. In order to get further insight into the effect of RSOC on electron

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Fig. 3. Angle- and subband-dependent transmissions through a barrier of wide $\Delta = 110$ nm and height $U = 145$ meV in SLG + R with $E_F = 80$ meV and $\lambda = 50$ meV. (a) Transmissions within $|\phi_1| < \phi_1^c = 17.83^\circ$ for the incident electron in subband $n = 1$. (b) Transmissions within $|\phi_2| < \phi_2^c = 39.6^\circ$ for the incident electron in subband $n = 2$.

Fig. 4. Angle- and subband-dependent transmissions through a barrier of wide $\Delta = 110$ nm and height $U = 200$ meV in SLG + R with $E_F = 80$ meV and $\lambda = 50$ meV. (a) Transmissions within $|\phi_1| < \phi_1^c = 28.71^\circ$ for the incident electron in subband $n = 1$. (b) Transmissions within $|\phi_2| < 90^\circ$ for the incident electron in subband $n = 2$. 

X.-D. Tan et al. / Superlattices and Microstructures 98 (2016) 473–491
Fig. 5. Transmission probabilities (a) $T_{11}^\text{tot}$, (b) $T_{12}$, (c) $T_{12}^\text{tot}$, (d) $T_{22}$, and (f) $T_{21}$ through a 110-nm-wide barrier as a function of incident angle $\phi$ and barrier height $U$ in SLG + R with $E_F = 80$ meV and $\lambda = 50$ meV.
transmissions, we depict the total transmission $T_{\text{tot}}$, for several different $\lambda$, as a function of the incident angle $\phi_n$ ($n = 1, 2$). As can be seen in Fig. 6, when $\lambda$ is very small it shows $T_{\text{tot}}^1 = T_{\text{tot}}^2$, which is in agreement with the previous results [4,34]. As we increase $\lambda$, $T_{\text{tot}}^1$ and $T_{\text{tot}}^2$ exhibit a big difference. This can be explained by the fact that when $\lambda \to 0$ the spin-up and the spin-down states of electrons are approximately degenerate, but when $\lambda$ is increased, the degeneracy is gradually removed. Thus, Klein tunneling becomes very complicated and subtle when the RSOC is noticeable.

4. Spin-resolved conductance

4.1. In the case of $S \to M$ transmission regime

For each transmitted electron in the state,

$$\psi = |n, k_n\rangle = \frac{1}{\sqrt{2LW(1 + \eta_n^2)}} \begin{bmatrix} i(-1)^{n-1}e^{-i2\phi_n} \\ \eta_n e^{-i\phi_n} \\ i(-1)^{n+1}e^{i\phi_n} \end{bmatrix} e^{i(k_n x + k_y y)} \quad (n = 1, 2),$$

the contribution to the current density along the $x$-direction is given by

$$j_{nx} = evF \sigma_x \psi = \frac{ehv_F^2}{\pi LW} \begin{bmatrix} \cos(\phi_n)k_n T_n^1 \\ \eta_1 \cos(\phi_1) T_n^1 + \eta_2 \cos(\phi_2) T_n^2 \\ 1 + \eta_1^2 \end{bmatrix},$$

where $n$ denotes the original subband in which an incident electron occupies, and

$$T_n^\pm = \frac{T_n^1 + T_n^2 \pm \eta_1 \cos(\phi_1) T_n^1 + \eta_2 \cos(\phi_2) T_n^2}{1 + \eta_1^2},$$

is the probability of an electron in subband $n$ throughout the barrier with the spin up (+) or the spin down (−) state of the real spin operator $S_y$ (for details see Appendix B). The total current density is determined by carriers with all possible values of $k_x$ and $k_y$. The number of electrons in range of $k \to k + dk$ is determined by the product of its state density and occupied possibility,

$$dn_{\text{elec}} = 2 \times f(E, \mu_e) \frac{LW}{4\pi^2} k_n dk_n d\phi_n = \frac{LW}{h^2 v_F^2} [2E - (-1)^n \lambda] f(E, \mu_e) dEd\phi_n,$$

where the factor 2 arises because $K$ and $K'$ valleys have the same contribution to the current density. Here
\[ f(E, \mu_z) = \frac{1}{1 + e^{(E-\mu_z)/k_BT}}, \tag{47} \]

is the Fermi-Dirac distribution. Note that there exist two reservoirs. The left reservoir injects carriers up to a quasi-Fermi-energy \( \mu_1 \) and the right reservoir emits carriers up to a quasi-Fermi-energy \( \mu_2 \). The two baths compete with each other and finally the electrons obtain equilibrium distribution with a certain chemical potential \( \mu_L \) and \( \mu_R \). Then the total net current density following from the left reservoir to the right reservoir is given by

\[ J^\pm_{nx} = e \int_{\phi_n^-}^{\phi_n^+} \cos(\phi_n) \, d\phi_n \int_{\mu_1}^{\mu_2} \frac{k_n}{\pi} \left[ f(E, \mu_L) - f(E, \mu_R) \right] T^n_{\mp} \, dE, \tag{48} \]

where \( \phi_n^\pm \) is the critical incidence angle, and \( T^n_{\mp} \) exists only if \( |\phi_n| < \phi_n^\pm \). The carrier densities can be characterized by the chemical potential \( \mu_L \) and \( \mu_R \), and their respective levels are between \( \mu_1 \) and \( \mu_2 \). If the bias is chosen to be small enough, then we can obtain the following approximation,

\[ f(E, \mu_L) - f(E, \mu_R) = \frac{\partial f}{\partial \mu} (\mu_L - \mu_R) = -\frac{\partial f}{\partial E} eV_0. \tag{49} \]

At very low temperature, we have

\[ \frac{\partial f}{\partial E} = \delta(E - E_F). \tag{50} \]

As a result, the spin-dependent current density along the x-direction is given by

\[ J^x_{nx} = e^2 \int_{\phi_n^-}^{\phi_n^+} k_n(E_F, \lambda) \cos(\phi_n) T^n_{\mp}(E_F, \lambda, U, \Delta, \phi_n) \, d\phi_n. \tag{51} \]

According to the definition of conductance \( (G = I/V) \), we obtain that

\[ I^x_{nx} = \int_0^W J^x_{nx} \, dy = J^x_{nx} W = G^n_{\mp} V_0. \tag{52} \]

Then, the effective conductance (conductance per unit length) is given by

\[ G^n_{\mp} = \frac{G^n_{\mp} W}{V_0}. \tag{53} \]

Thus, we explicitly obtain the spin-dependent conductance in the \( S \rightarrow M \) transmission regime,

\[ G^n_{\mp} = e^2 \frac{1}{\hbar} \int_{\phi_n^-}^{\phi_n^+} k_n(E_F, \lambda) \cos(\phi_n) T^n_{\mp}(E_F, \lambda, U, \Delta, \phi_n) \, d\phi_n. \tag{54} \]

Since we are interested in the states of outgoing electrons, we define the conductance for outgoing spin up electrons as

\[ G^{S\rightarrow M}_1 = G^+_1 + G^+_2, \tag{55} \]

and similarly,

\[ G^{S\rightarrow M}_1 = G^-_1 + G^-_2. \tag{56} \]

Then the total conductance in the \( S \rightarrow M \) transmission regime is given by
4.2. In the case of \(S \rightarrow S\) transmission regime

In this case, the single electron current density along the \(x\)-direction is given by

\[
\tilde{j}_{nx} = \frac{e \hbar^2 k_n \tilde{T}_n^{\pm}}{\pi L W[2E - (-1)^n \lambda]},
\]

with

\[
\tilde{T}_n^{\pm} = \frac{\tilde{T}_{nn}^{\pm}}{2} \pm (-1)^n \frac{\eta_n \tilde{T}_{nn}}{1 + \eta_n^2}.
\]

The number of normally incident electrons in range of \(k_x \rightarrow k_x + dk_x\) is given by

\[
d\tilde{n}_{\text{elec}} = 2 \times \frac{L}{2\pi} f(E, \mu_x) dk_n = \frac{2\pi L[2E - (-1)^n \lambda]}{\hbar^2 v_F^2 k_n} f(E, \mu_x) dE.
\]

The total net current density is calculated as

\[
\tilde{j}_{nx} = \frac{e^2}{h} \times \frac{2V_0}{W} \tilde{T}_n^{\pm}(E_F, \lambda, U, \Delta).
\]

Then, at very low temperature the current intensity is given by

\[
G_{\text{tot}}^{S \rightarrow M} = G_1^{S \rightarrow M} + G_1^{S \rightarrow M}.
\]
Thus, the spin-dependent conductance in the $S \rightarrow S$ transmission regime is given by

$$\tilde{\mathcal{G}}_n^\pm = \frac{e^2}{h} \times 2T_n^\pm(E_F, \lambda, U, \Delta).$$

Conformance to definitions in Eqs. 55–57, it is clear to give the following expressions

$$G_{\uparrow\downarrow}^{S\rightarrow S} = \begin{cases} \tilde{\mathcal{G}}_1^\uparrow, & (0 < E_F < \lambda, U > 0) \\ \tilde{\mathcal{G}}_1^\downarrow, & (E_F > \lambda, E_F - \lambda < U < E_F) \\ \tilde{\mathcal{G}}_2^\downarrow, & (E_F > \lambda, E_F < U < E_F + \lambda) \end{cases}$$

$$G_{\uparrow\downarrow}^{S\rightarrow S} = \begin{cases} \tilde{\mathcal{G}}_1^\downarrow, & (0 < E_F < \lambda, U > 0) \\ \tilde{\mathcal{G}}_1^\uparrow, & (E_F > \lambda, E_F - \lambda < U < E_F) \\ \tilde{\mathcal{G}}_2^\uparrow, & (E_F > \lambda, E_F < U < E_F + \lambda) \end{cases}$$

$$G_{\text{tot}}^{S\rightarrow S} = G_{\uparrow\downarrow}^{S\rightarrow S} + G_{\uparrow\downarrow}^{S\rightarrow S}.$$
4.3. Numerical results and discussions

We are now in a position to discuss the spin-resolved conductance. First, we consider the nontrivial case of \( 0 < E_F < \lambda \). As can be seen in Fig. 7, the total conductance \( G_{\text{tot}} \) is almost dominated by \( G_Y \). At lower barrier, \( G_{\text{tot}} \) approximately keeps a maximal constant \( \frac{2e^2}{h} \). When the barrier lies in the interval of \( E_F < U < E_F + \lambda \), \( G_{\text{tot}} \) is equal to zero. While for the remaining interval of \( U \), the spin-resolved conductance displays oscillating behaviors. These remarkable features show that fabrication of a graphene-based FET with a noticeable on-off ratio is feasible in the presence of a sizeable RSOC (\( \lambda > E_F \)).

Next, we discuss the conductance for the case of \( E_F > \lambda \). We can see in Fig. 8 that for a given \( \lambda = 50 \) meV the spin-resolved conductance \( G_Y \) (\( G_L \)) is monotone decreasing in the range of \( 0 < U < E_F - \lambda \). When we increase \( U \) to the interval of \( E_F - \lambda < U < E_F + \lambda \), spin-up electrons are completely reflected and only spin-down electrons have contribution to conductance. While for the range of \( E_F < U < E_F + \lambda \), it is just the opposite. Further increasing \( U \), the conductance \( G_Y \) (\( G_L \)) displays oscillating behaviors. For \( \lambda = 50 \) meV, the total conductance \( G_{\text{tot}} \) does not exist a zero-gap along the \( U \)-axis. We also compared \( G_{\text{tot}} \) for several different \( \lambda \), as shown in the inset of Fig. 8. We can see that when \( U \) falls into the region \( E_F - \lambda < U < E_F + \lambda \), \( G_{\text{tot}} \) is low and fluctuates with \( U \). Out of this region, \( G_{\text{tot}} \) tends to decrease with increasing \( \lambda \). More detailed calculation shows that in the range of \( E_F - \lambda < U < E_F + \lambda \) the maximal \( G_{\text{tot}} \) is no more than \( 2e^2/h \), while the minimum is not zero except some special values of \( U \). Thus, it is difficult to achieve considerable on-off ratios for graphene-based FETs in the case of \( E_F > \lambda \).
5. Spin polarization in the transmitted region

According to the above analysis, the transmitted electrons can be spontaneously spin polarized along the y-direction. So, it is necessary to quantify the spin polarization of electrons in the transmitted region. Following Ref. [35], the spin polarization can be defined as

$$P = \frac{G_1 - G_2}{G_1 + G_2}.$$  \hspace{1cm} (67)

In the case of $S \rightarrow S$ transmission regime, it is easy to obtain

$$P = \frac{G_1^{S \rightarrow S} - G_2^{S \rightarrow S}}{G_1^{S \rightarrow S} + G_2^{S \rightarrow S}} = \begin{cases} \frac{2\sqrt{E_F(E_F + \lambda)}}{2E_F + \lambda}, & (0 < E_F < \lambda, \ U > 0) \\ \frac{2\sqrt{E_F(E_F - \lambda)}}{2E_F - \lambda}, & (E_F > \lambda, \ E_F - \lambda < U < E_F) \\ \frac{2\sqrt{E_F(E_F + \lambda)}}{2E_F + \lambda}, & (E_F > \lambda, \ E_F < U < E_F + \lambda) \end{cases}$$  \hspace{1cm} (68)

which is a constant independent of $U$ when $E_F$ and $\lambda$ are given. For $E_F = 30$ meV and $\lambda = 50$ meV, we have $P = -0.8907$. When $E_F > \lambda$, the high spin polarization also occurs in the range of $EF - \lambda < U < EF + \lambda$. Moreover, the direction of spin polarization is opposite near the critical point $U = EF$. For example, when we fix $E_F = 80$ meV and $\lambda = 50$ meV, we have $P = -0.9712$ for 30 meV $< U < 80$ meV, and $P = 0.8907$ for 80 meV $< U < 130$ meV (see Fig. 9). This remarkable result indicates that one can realize spin switches in the region $EF - \lambda < U < EF + \lambda$. When $U$ is located in the range of $0 < U < EF - \lambda$, the spin polarization increases with increasing $U$ on the whole. In the range of $U > EF + \lambda$, spin polarization is weak and displays oscillating behaviors with increasing $U$. By comparing the spin polarization for several different $\lambda$, it clearly shows that RSOC plays a significant role in spin polarized transport in SLG + R.

6. Potential applications

Graphene PN and PNP junctions provide an effective platform for investigating Klein tunneling [4] and other relevant quantum electrodynamics phenomena. They are also the most important basic building blocks in spintronics devices. Based on the above discussions, we propose a conceptive graphene-spin device. Most works adopted the top-gated graphene-based device. However, impurities and organic residue caused by fabrication processes degrade the quality of graphene layer [36,37]. In Ref. [38], it presents a new method to fabricate a graphene device with neither deposition of dielectric material on the graphene nor electron-beam irradiation. Unlike top-gated devices, in this scheme the pre-defined local gate (poly-silicon) is embedded into a SiO2 substrate, as shown in Fig. 10. This local gate is used to create a tunable potential barrier, and the back gate controls the Fermi lever of SLG but does not affect the carrier density in the local-gate region [38]. On the other hand, the considerable SOC is a crucial element in the development of graphene-based spin FETs and realization of topological insulators. To enhance the SOC strength, one can deposit the graphene sheet on some special substrates such as tungsten disulfide (WS2) and Au/Ni(111). As reported in Refs. [18,19], strong SOC can be induced in these systems. By tuning local and back gate voltages, both control of barrier height and Fermi lever in SLG can be achieved, thereby manipulating spin transport properties of carriers. According to the detailed theoretical analyses in the present study, we think that this device may promisingly qualify as a spin FET. At least, it might realize the functions such as spin filtration, spin switch, and electron beam collimation.

7. Conclusions

In summary, we investigated the spin transport properties in SLG + R. The transmission coefficients through a potential barrier were obtained analytically. Results show that Klein tunneling in SLG + R and BLG exhibits completely different behaviors. The tunneling at normal incidence in BLG shows a perfect reflection, while in SLG + R it displays more dramatic behaviors. In the $S \rightarrow S$ transmission regime that mainly occurs in the case of $0 < E_F < \lambda$, only normally incident electrons have capacity to pass through the barrier in SLG + R. Moreover, the tunneling between different spin-subbands is forbidden in this regime, and consequently outgoing electrons are highly spin polarized along y-direction. As reported in Ref. [4], in the absence of RSOC, the barrier always remains perfectly transparent for electrons close to the normal incidence. However, in the presence of RSOC, this perfect tunneling is not always allowed unless the resonance conditions are satisfied. In the $S \rightarrow M$ transmission regime that only exists in the case of $E_F > \lambda$, the electron transmission depends not only on subbands but also strongly on incident angles. In this regime, the tunneling could happen at not only normal but also oblique incidence. Based on the transmission coefficients, we also derived spin-resolved conductance analytically. The most significant finding is that when $0 < E_F < \lambda$ the conductance appears a gap in the interval of $E_F < U < E_F + \lambda$. This remarkable features indicates that
fabrication of a graphene-based FET with a high on-off ratio is feasible if $0 < E_F < \lambda$. In the case of $E_F > \lambda$, the conductance decays with increasing $U$ if $0 < U < E_F - \lambda$. When $E_F - \lambda < U < E_F + \lambda$, the electrons are highly spin polarized, and spin switch effect emerges near the critical point $U = E_F$. We also found that the conductance displays oscillation behaviors in this interval as well as the interval of $U > E_F + \lambda$. For $E_F > \lambda$, except some special values of $U$, it is difficult to pinch off the conducting channel, which limits achievable on-off ratios for graphene-based FETs in this case, but it can serve as a spin switch.

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Appendix A. angle- and subband-dependent tunneling probabilities

Based on the boundary conditions, $\psi_I(x = 0) = \psi_H(x = 0)$ and $\psi_II(x = \Delta) = \psi_III(x = \Delta)$, we obtain the following equation set

$$
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\beta_1 \\
\beta_2
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
\epsilon_1 \\
\epsilon_2
\end{bmatrix},
$$

where the specific parameters are listed below

$$
A = 
\begin{bmatrix}
e^{i(q_1\Delta - 2\phi_1)} & e^{-i(q_1\Delta - 2\phi_1)} & -e^{i(q_2\Delta - 2\phi_2)} & -e^{-i(q_2\Delta - 2\phi_2)} \\
\xi_{11}e^{i(q_1\Delta - \phi_1)} & -\xi_{11}e^{-i(q_1\Delta - \phi_1)} & \xi_{21}e^{i(q_2\Delta - \phi_2)} & -\xi_{21}e^{-i(q_2\Delta - \phi_2)} \\
\xi_{12}e^{i(q_1\Delta - \phi_1)} & -\xi_{12}e^{-i(q_1\Delta - \phi_1)} & \xi_{22}e^{i(q_2\Delta - \phi_2)} & -\xi_{22}e^{-i(q_2\Delta - \phi_2)} \\
e^{-i(q_1\Delta)} & e^{i(q_1\Delta)} & e^{-i(q_2\Delta)} & e^{i(q_2\Delta)}
\end{bmatrix},
$$

$$
B = 
\begin{bmatrix}
0 & 0 & -e^{i(k_1\Delta - 2\phi_1)} & e^{i(k_2\Delta - 2\phi_2)} \\
0 & 0 & -\eta_1e^{i(k_1\Delta - \phi_1)} & -\eta_2e^{i(k_2\Delta - \phi_2)} \\
0 & 0 & -e^{i(k_1\Delta - 2\phi_1)} & e^{i(k_2\Delta - 2\phi_2)} \\
0 & 0 & -\eta_1e^{i(k_1\Delta - \phi_1)} & -\eta_2e^{i(k_2\Delta - \phi_2)}
\end{bmatrix},
$$

$$
C = 
\begin{bmatrix}
e^{-i2\phi_1} & e^{i2\phi_1} & -e^{-i2\phi_2} & e^{i2\phi_2} \\
\xi_{11}e^{-i\phi_1} & -\xi_{11}e^{i\phi_1} & \xi_{21}e^{-i\phi_2} & -\xi_{21}e^{i\phi_2} \\
\xi_{12}e^{-i\phi_1} & -\xi_{12}e^{i\phi_1} & \xi_{22}e^{-i\phi_2} & -\xi_{22}e^{i\phi_2} \\
1 & 1 & 1 & 1
\end{bmatrix},
$$

$$
D = 
\begin{bmatrix}
-e^{-i2\phi_1} & e^{i2\phi_1} & 0 & 0 \\
e^{i\phi_1} & \eta_1e^{i\phi_1} & \eta_2e^{i\phi_2} & 0 \\
e^{i\phi_2} & -\eta_1e^{i\phi_1} & -\eta_2e^{i\phi_2} & 0 \\
-1 & -1 & 0 & 0
\end{bmatrix},
$$

$$
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\beta_1 \\
\beta_2
\end{bmatrix},
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\beta_1 \\
\beta_2
\end{bmatrix},
$$

$$
\epsilon_1 = \frac{1}{\sqrt{2(1 + \eta_1^2)}} \begin{bmatrix}
1 \\
\eta_1e^{-i\phi_1} \\
\eta_1e^{i\phi_1} \\
1
\end{bmatrix},
\epsilon_2 = \frac{1}{\sqrt{2(1 + \eta_2^2)}} \begin{bmatrix}
-e^{-i2\phi_2} \\
\eta_2e^{-i\phi_1} \\
-\eta_2e^{i\phi_1} \\
1
\end{bmatrix}.
$$

Using the Cramer’s rule and block matrix skill, straightforward calculation then gives the expressions of the transmission coefficients

$$
T_{11} = \frac{\det(H_{11} - CA^{-1}M)^2}{\det(D - CA^{-1}B)},
$$

$$
T_{12} = \frac{\eta_2 \cos(\phi_2)}{\eta_1 \cos(\phi_1)} \frac{\det(H_{12} - CA^{-1}F)^2}{\det(D - CA^{-1}B)},
$$

$$
T_{21} = \frac{\eta_1 \cos(\phi_1)}{\eta_2 \cos(\phi_2)} \frac{\det(H_{21} - CA^{-1}E)^2}{\det(D - CA^{-1}B)},
$$

$$
T_{22} = \frac{\det(H_{22} - CA^{-1}G)^2}{\det(D - CA^{-1}B)}.
$$
\[ T_{21} = \frac{\eta_1 \cos(\phi_1)}{\eta_2 \cos(\phi_2)} \left| \frac{\det(H_{21} - CA^{-1}M)}{\det(D - CA^{-1}B)} \right|^2, \]  
\[ T_{22} = \left| \frac{\det(H_{22} - CA^{-1}F)}{\det(D - CA^{-1}B)} \right|^2, \]  
(A10)

with

\[ H_{11} = \begin{bmatrix} -e^{i2\phi_1} & e^{i2\phi_2} & e^{-i2\phi_1} & 0 \\ \eta_1 e^{i\phi_1} & \eta_2 e^{i\phi_2} & \eta_1 e^{i\phi_1} & 0 \\ \eta_1 e^{i\phi_1} & -\eta_2 e^{i\phi_2} & \eta_1 e^{i\phi_1} & 0 \\ -1 & -1 & 1 & 0 \end{bmatrix}, \quad H_{12} = \begin{bmatrix} -e^{i2\phi_1} & e^{i2\phi_2} & e^{-i2\phi_1} & 0 \\ \eta_1 e^{i\phi_1} & \eta_2 e^{i\phi_2} & \eta_1 e^{i\phi_1} & 0 \\ \eta_1 e^{i\phi_1} & -\eta_2 e^{i\phi_2} & \eta_1 e^{i\phi_1} & 0 \\ -1 & -1 & 1 & 0 \end{bmatrix}, \]  
(A12)

\[ H_{21} = \begin{bmatrix} -e^{i2\phi_1} & e^{i2\phi_2} & -e^{-i2\phi_1} & 0 \\ \eta_1 e^{i\phi_1} & \eta_2 e^{i\phi_2} & \eta_1 e^{i\phi_1} & 0 \\ \eta_1 e^{i\phi_1} & -\eta_2 e^{i\phi_2} & \eta_1 e^{i\phi_1} & 0 \\ -1 & -1 & 1 & 0 \end{bmatrix}, \quad H_{22} = \begin{bmatrix} -e^{i2\phi_1} & e^{i2\phi_2} & -e^{-i2\phi_1} & 0 \\ \eta_1 e^{i\phi_1} & \eta_2 e^{i\phi_2} & \eta_1 e^{i\phi_1} & 0 \\ \eta_1 e^{i\phi_1} & -\eta_2 e^{i\phi_2} & \eta_1 e^{i\phi_1} & 0 \\ -1 & -1 & 1 & 0 \end{bmatrix}, \]  
(A13)

\[ M = \begin{bmatrix} 0 & 0 & 0 & e^{i(k_D - 2\phi_1)} \\ 0 & 0 & 0 & -\eta_2 e^{i(k_D - 2\phi_1)} \\ 0 & 0 & \eta_2 e^{i(k_D - 2\phi_1)} & 0 \\ 0 & 0 & -e^{i(k_D - \Delta)} & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 0 & 0 & -e^{i(k_D - 2\phi_1)} & 0 \\ 0 & 0 & -\eta_1 e^{i(k_D - \phi_1)} & 0 \\ 0 & 0 & -\eta_1 e^{i(k_D - \phi_1)} & 0 \\ 0 & 0 & -e^{i(k_D - \Delta)} & 0 \end{bmatrix}. \]  
(A14)

Here, the inverse of the matrix \( A \) is calculated as

\[ A^{-1} = \begin{bmatrix} a & b & d & a \\ a^* & -b^* & -d^* & a^* \\ p & -m & -g & -p \\ p^* & m^* & g^* & p^* \end{bmatrix}, \]  
(A15)

where the parameters are given by

\[ d = a \omega, \quad b = a \omega^*, \]  
(A16)

with \( \omega = \sec(\theta_1) e^{-i(\eta_1 \tilde{D} - \phi_1) / 4} \) and \( \omega = \cos(\theta_1) / \xi_{15} + i \sin(\theta_1) / \xi_{25} \), and

\[ m = p \chi, \quad g = -p' \chi^*, \]  
(A17)

with \( p = -\sec(\theta_2) e^{i(\eta_2 \tilde{D} - \phi_2) / 4} \) and \( \chi = i \sin(\theta_1) / \xi_{15} + \cos(\theta_2) / \xi_{25} \).

Appendix B. spin-resolved transmission probabilities

In the basis of \( \{|u_{A1}\}, \{|u_{A2}\}, \{|u_{B1}\}, \{|u_{B2}\}\} \), the real spin matrix \( S_y \) reads

\[ \begin{bmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{bmatrix}. \]  
(B1)

Its eigenstates for the eigenvalue \( s_y = 1 \) are

\[ |\uparrow_1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} i \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad |\uparrow_2\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ i \\ -1 \end{bmatrix}, \]  
(B2)

and for the eigenvalue \( s_y = -1 \) are
\[ |\downarrow_1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} i \\ 1 \\ 0 \end{bmatrix}, \quad |\downarrow_2\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ i \end{bmatrix}. \]  

(B3)

When a transmitted electron in the state
\[ \psi = |n, k_n\rangle = \frac{1}{\sqrt{2LW(1 + \eta_n^2)}} \begin{bmatrix} i(-1)^{n+1} e^{-i2\phi} \\ \eta_n e^{-i\phi} \\ i \eta_n e^{-i\phi} \end{bmatrix} e^{i(k_n x_k y)}, \quad (n = 1, 2), \]  

one can detect the spin-up electron occupying subband \( n \) with a probability,
\[ P_n^+ = |\langle \uparrow |n, k_n\rangle|^2 + |\langle \uparrow |n, k_n\rangle|^2 = \frac{1}{2} + (-1)^n \frac{\eta_n \cos(\phi_n)}{1 + \eta_n^2}, \]  

and the spin-down electron with a probability,
\[ P_n^- = |\langle \downarrow |n, k_n\rangle|^2 + |\langle \downarrow |n, k_n\rangle|^2 = \frac{1}{2} - (-1)^n \frac{\eta_n \cos(\phi_n)}{1 + \eta_n^2}. \]  

(B4)

Thus, in the \( S \rightarrow M \) transmission regime, the probability that one observes an electron originally occupying subband \( n \) and going out the barrier with spin up (\( + \)) or spin down (\( - \)) state is given by
\[ T_n^\pm = T_{n1} P_n^+ + T_{n2} P_n^- = \frac{T_{n1} + T_{n2}}{2} \frac{\eta_1 \cos(\phi_1) T_{n1} \eta_2 \cos(\phi_2) T_{n2}}{1 + \eta_1^2 + \eta_2^2}. \]  

(B7)

Similarly, one can obtain the spin-dependent probability in the \( S \rightarrow S \) transmission regime, i.e.,
\[ \tilde{T}_n^\pm = \tilde{T}_{nn} P_n^+ = \frac{\tilde{T}_{nn} \eta_1 \cos(\phi_1) \tilde{T}_{nn}}{1 + \eta_1^2}. \]  

(B8)

References